

# The leaching number: Its definition and use in determining the performance of leaching reactors and autoclaves

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## Abstract

Leaching is a critical operation in hydrometallurgy. The performance of the leaching sections of a processing plant usually has a significant impact on the performance of the plant as a whole. In addition, leaching reactors are usually complex, involving multiple reactions and involving gas, liquid and solid phases. In spite of the importance of leaching reactors, the understanding, and hence the design, of continuous leaching is based on concepts originating from batch leaching experiments. Many of these concepts, such as “rate-controlling step” and “pseudo-steady state” are inappropriately used to interpret the reactor performance or to design leaching reactors. In this regard, the leaching number is derived. The leaching number is defined as  $\bar{t}r_s/\bar{l}$ , where  $\bar{t}$  is the mean residence time,  $r_s$  is the rate of shrinkage of the particles and  $\bar{l}$  is the mean particle size. In addition to the simplicity of the definition of this dimensionless leaching number, it is shown in this paper that it is the most important parameter in understanding the performance of a leaching reactor.

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## 1. Introduction

Leaching reactors are unique amongst tank operations. They generally involve three phases, one of which, the solids, changes in size. The closest operation to leaching is crystallisation, in which the solid phase grows in size rather than shrinks. In spite of the great strides with which the field of the design of chemical reactors has moved, leaching reactors are more poorly understood, and the design is generally unsophisticated. In the design process, emphasis is placed mainly on the leaching conditions and chemistry, with little regard for the reactor engineering of this operation.

The lack of a thorough design method has undoubtedly resulted in excessive spending on capital and increased operating costs due to poorer control. For

example, it is clear from the data presented for the operation of the zinc pressure leaching reactor at Trail, British Columbia, that this commercial unit could operate at up to 200% above its design capacity (Crundwell and Bryson, 1992; Martin and Jankola, 1976). Since the pressure autoclaves contribute as much as 30% to the total capital expenditure for a hydrometallurgy plant, such gross over-estimation of autoclave capacity could result in poor economic and engineering decision-making.

The factors that need to be accounted for in a thorough design procedure are:

- solids throughput,
- particle size distribution,
- retention time,
- reaction kinetics,
- slurry density,
- lixiviant throughput,
- circuit configuration.

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The earliest method of design for leaching reactors arose in the late fifties and early 60s. It was a graphical design procedure based on the assumption that leaching reactions are essentially the same as homogeneous reactions (Caddel and Hurt, 1951). This assumes that the particle size and particle size distribution have no influence on the performance of the reactor. In spite of the increased knowledge and computer tools currently available, this is probably still the most used technique.

The discovery that the results of batch leaching experiments are described by the shrinking-core model lead to the incorporation of the shrinking-core model into the design procedure (Bartlett, 1971; Henien and Biegler, 1988; Pritzker, 1993). This design procedure attempted to account for shrinking surface area without accounting for the continuous feeding and removal of particles of different sizes from the system. In other words, the change in surface area was accounted for while it was still assumed that leaching reactors were homogenous, not heterogeneous. These models are conceptually and practically without merit.

Leaching systems are not homogenous: the particles react, and the surface area at which reaction takes place changes both with residence time and with particle size distribution. Neither of the approaches discussed above could describe this situation. However, two different methods arose at about the same time to describe heterogeneous reactors. These were the segregated-flow model (Crundwell, 1994; Levenspiel, 1972; Zwietering, 1958), and the population balance model, described by Hulburt and Katz (1964) and Randolph and Larson (1971). The segregated-flow model and the population balance are not equivalent either conceptually or numerically. Crundwell (1994) has explored their differences, and has discussed the subtle assumptions that differentiate them.

The aim of this paper is to define the leaching number, show how the same dimensionless number arises

for both of these two models and to demonstrate its value in understanding continuous leaching operations. The leaching number is defined here as the following:

$$N_L = \frac{r_s \bar{t}}{\bar{\ell}} \quad (1)$$

where  $\bar{t}$  is the mean residence time,  $r_s$  is the rate of shrinkage of the particles (leaching kinetics) and  $\bar{\ell}$  is the mean particle size. The leaching rate,  $r_s$ , has units of metres per second.

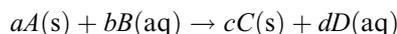
We will demonstrate that in order to achieve a particular conversion in a reactor, there are only three variables to manipulate: (i) the residence time, (ii) the particle size, and (iii) the intrinsic leaching rate (through temperature).

This paper is structured as follows. The batch leaching kinetics, and the segregated-flow and the population balance models of continuous reactors will be described in order to expose the leaching number as the controlling variable. The significance and importance of the leaching number will be discussed. A sensitivity analysis will be used to provide further insight into the behaviour of leaching reactors.

The primary goal of this paper is not models, not numbers, but knowledge. It is the knowledge imbedded in the leaching number that will enable better understanding and design leaching reactors.

## 2. The leaching number in kinetics

The dissolution of mineral particles by a lixiviant or reactant in solution results in the shrinkage of the particle. For a general reaction, such as



there is the possibility that a solid reaction product forms on the surface of the unreacted material. This is illustrated in Fig. 1. For this reason, the standard model

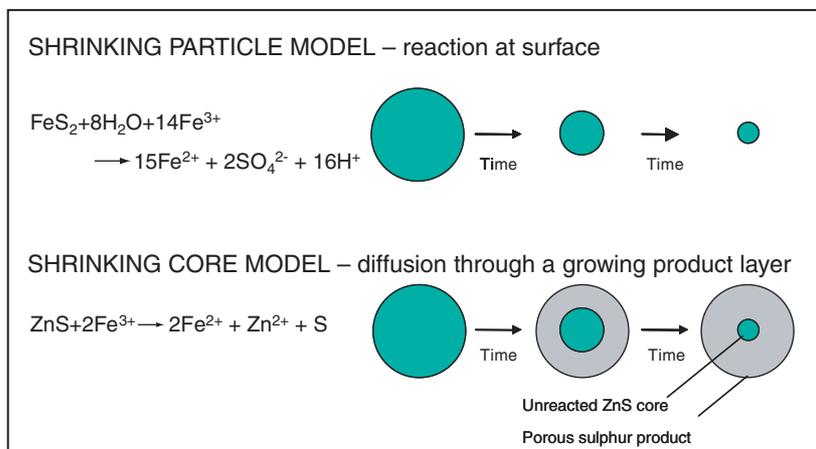


Fig. 1. A schematic illustration of the situations described by the shrinking-core model and the shrinking particle model.

is referred to as the shrinking-core model, since it is the core of unreacted material surrounded by product that shrinks in size. If no reaction product forms on the surface, then the model is referred to as the shrinking particle model.

The shrinking-core model is based purely on the geometry of the change in surface area with change in size. The model assumes nothing of the intrinsic mechanism of reaction at the surface of the particle.

The shrinking-core model is given by

$$\frac{d\ell}{dt} = r_s = \frac{-2a/b}{\rho \left[ \frac{1}{k_s[B]^n} + \frac{1}{2D_B} (\ell - \ell^2/L) + \frac{1}{k_f} (\ell/L)^2 \right]} \quad (2)$$

If the rate of shrinkage is controlled by reaction at the surface, then

$$\frac{d\ell}{dt} = r_s = \frac{-2a}{b\rho} k_s [B]^n \quad (3)$$

If and only if  $r_s$  is constant, Eq. (3) can be integrated directly to yield the well-known form of the shrinking-core model:

$$1 - (1 - X)^{1/3} = \frac{r_s t}{L} \quad (4)$$

where  $X$  is the conversion achieved after time  $t$ ,  $r_s$  is the rate of dissolution at the surface in units of length per time (similar to a corrosion rate), and  $L$  is the initial particle size. The conversion is related to size by  $X = 1 - \ell^3/L^3$ .

The intrinsic rate of reaction, given by  $r_s$ , describes the mechanism of reaction at the surface. If the reaction is a simple reaction of order  $n$  and activation energy  $E_a$ , then the rate of reaction is given by

$$r_s = kC^n \exp(-E_a/RT) \quad (5)$$

It has often been found that the rate of leaching reactions is half order ( $n = 0.5$ ), in keeping with the prediction of the electrochemical model of leaching, and that the activation energy is above 40 kJ/mol.

It is clear that the right-hand side of Eq. (4) is a dimensionless number of the same form as the leaching number. This is the form in which the batch leaching number is defined:

$$N_L = \frac{r_s t}{L} \quad (6)$$

The importance of the leaching number, given in Eq. (6), is this: in order to increase the extent of leaching, given by  $X$ , only three factors can be varied. Either the rate must be increased, the initial particle size decreased, or the leaching time lengthened.

The shrinking-core mode is well known. However, models for continuous leaching, although well established, have not gained wide acceptance in the design of leaching reactors. It is in this regard that the leaching

number will add knowledge and insight to the mathematics of leaching.

### 3. The leaching number in continuous leaching

The smaller a particle, the faster it dissolves due to the increase in surface area that is available per unit mass. It is self-evident that a successful model of continuous leaching must be able to account for the effect of particle size on the performance of the reactor. In addition, particles with a range of sizes are fed to a tank reactor, and, due to the conditions of mixing, they may leave the reactor after having spent different times in the reactor. Since the reaction is dependent on surface area, an effective model of the leaching must account for the change in both the particle size and the particle size distribution.

Models of the leaching of particulates in continuous reactors fall into two camps: the segregated-flow model and the population balance model. The underlying assumptions that distinguish these models are subtle, and relate to the degree of micro-mixing in the reactor. Crundwell (1994) derived the population balance and the segregated-flow models from the number balance, showed their relationship to one another, and demonstrated their numerical differences. Both models will be discussed with a view to determining the importance of the leaching number.

#### 3.1. Population-balance model

The mass balance for a component in a continuous well-stirred tank reactor is written as:

$$QC_{in} = QC_{out} - rV \quad (7)$$

where  $Q$  is the volumetric flow rate,  $C$  is the concentration,  $r$  is the rate of formation, and  $V$  is the volume of the tank.

Despite the familiarity of this formulation, it contains an assumption that is not well known. The model assumes that material is well mixed *on a molecular scale*. This condition is known as maximum mixedness. Crundwell (1994) showed that the population balance is the particulate equivalent of the component mass balance given in Eq. (7).

The derivation of the population balance is illustrated in Figs. 2 and 3. A typical tank reactor is shown in Fig. 2, which indicates the elements that need to be calculated in order to determine the performance of the reactor. The change in particle size distribution from the inlet to the outlet is also shown. This difference in distribution represents the material that is dissolved as a result of the leaching reaction.

A single size class of the distribution of particle sizes in the tank, which is the same as that leaving the tank, is shown in Fig. 3. Particles can come into this size class in

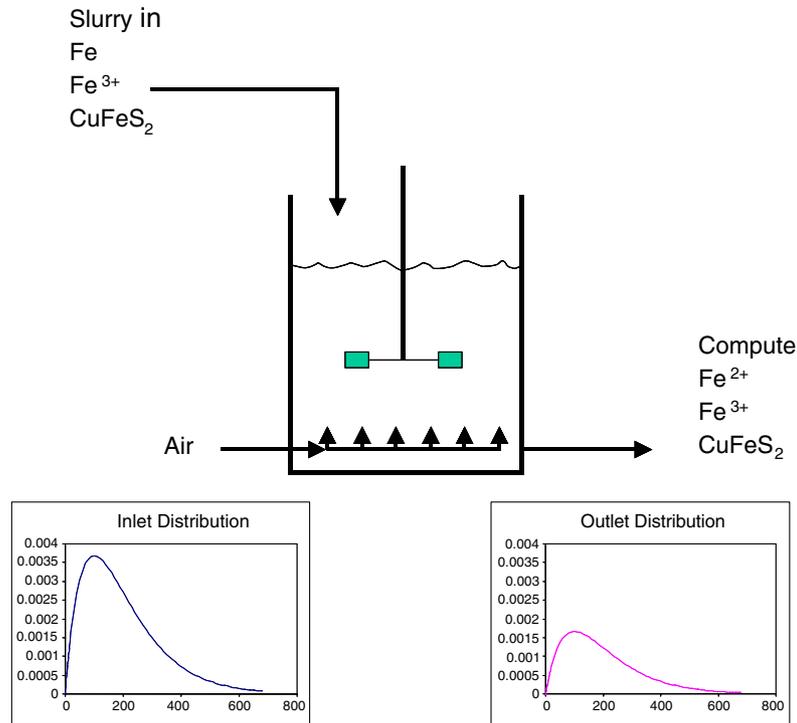


Fig. 2. A typical tank reactor. In order to determine the performance of the reactor, the outlet concentrations and the outlet size distribution must be calculated.

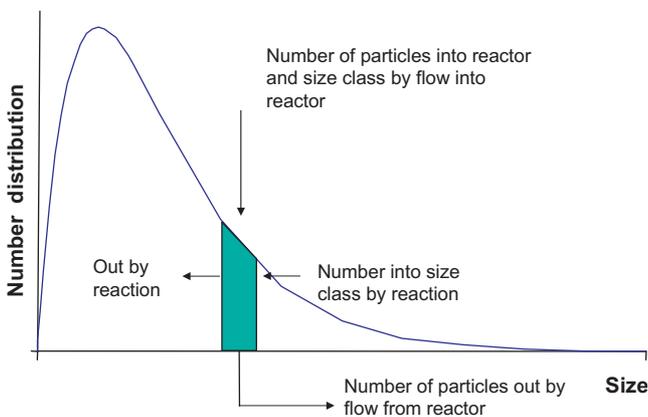


Fig. 3. The number balance over a size class in the reactor.

two ways: (i) particles with size in that class flow into the reactor from the feed, or (ii) particles that are already in the reactor shrink by reaction into that size class. A similar statement can be made about particles leaving the size class. As a result, we can write the population balance as:

$$\begin{aligned}
 &(\text{Number of particles into the reactor and} \\
 &\text{into the size class}) = (\text{Number of particles out of} \\
 &\text{reactor and out of the size class}) + (\text{Net number of} \\
 &\text{particles out of size class due to shrinkage by reaction})
 \end{aligned}
 \tag{8}$$

Eq. (8) is in the familiar form of a material balance given in Eq. (7) that is, in = out + reacted. It is important to note that Eq. (8) is an accounting system in which the number of particles is conserved. This is a familiar concept, since Eq. (7) is based on moles, which is defined as a multiple of Avogadro's number of molecules (particles). Of course, Eq. (8) has only been written for a single class of particle size. The derivation of the population balance from Eq. (8) by the methods of calculus has been described by Crundwell (1994) and others, and for a single tank reaction, it is given by

$$Qn_{in} = Qn_{out} - V \frac{dr_s n_{out}}{d\ell}
 \tag{9}$$

where  $r_s$  is the linear rate of particle shrinkage in m/s,  $V$  is the tank volume, and  $Q$  is the slurry feed rate to the tank. The terms  $n_{in}$  and  $n_{out}$  represent the inlet and outlet size functions, respectively.

The term on the left-hand side of Eq. (9) represents the material coming into the reactor with size  $\ell$ , while the first term on the right-hand side represents the material leaving the reactor with size  $\ell$ , and the second term on the right-hand side represents the change in particle size as a result of reaction. The rate of reaction is dependent on particle size, which has been made explicit in the term  $dr_s n/d\ell$ . Eq. (9) is a linear first-order differential equation that can be easily solved by analytical and numerical methods.

The model is completed by determining the conversion from the change in the size distribution. The

conversion of mineral in the leaching reactor is calculated from a determination the mass of mineral leaving the reactor from the size distribution

$$X = 1 - \frac{\int_0^\infty \ell^3 n_{\text{out}}(\ell) d\ell}{\int_0^\infty \ell^3 n_{\text{in}}(\ell) d\ell} \quad (10)$$

The mass or mole flow rate of unreacted mineral, say sphalerite, leaving the reactor calculated from a definition of the conversion, is given by

$$\{\text{ZnS}\}_{\text{out}} = \{\text{ZnS}\}_{\text{in}}(1 - X) \quad (11)$$

The rearrangement of Eq. (9) into dimensionless form yields the following equation:

$$n_{\text{in}}(\ell^*) = n_{\text{out}}(\ell^*) - \frac{d\left(\frac{\bar{t}}{\bar{\ell}} n(\ell^*)\right)}{d\ell^*} \quad (12)$$

where  $\bar{t}$  is the mean residence time,  $\bar{\ell}$  is the mean particle size in the feed, and  $\ell^*$  is  $\ell/\bar{\ell}$ . The terms  $n_{\text{in}}(\ell^*)$  and  $n_{\text{out}}(\ell^*)$  represent the dimensionless forms of the inlet and outlet size functions, respectively.

The substitution of the dimensionless leaching number into Eq. (12) yields:

$$n_{\text{in}}(\ell^*) = n_{\text{out}}(\ell^*) - \frac{dN_L n(\ell^*)}{d\ell^*} \quad (13)$$

Clearly, the leaching number is the only parameter affecting the performance of the leaching reactor. The significance of the leaching number is paramount: in order to increase the conversion achieved in the reactor, the leaching number must be increased. This will be discussed more fully later in the paper.

### 3.2. Segregated-flow model

In the late nineteen fifties, Danckwerts (1958) and Zwietering (1958) formulated an alternative mass balance to that described in Eq. (7). This is given by

$$C_{\text{out}} = \int_0^\infty C_{\text{batch}}(C_{\text{in}}, t) E(t) dt \quad (14)$$

$E(t)$  is the residence-time distribution, and represents the probability of a element of fluid exiting the reactor after time  $t$ .  $C_{\text{batch}}$  is the concentration achieved in a batch reactor after time  $t$  starting at the initial concentration of the inlet stream,  $C_{\text{in}}$ , and is given by

$$C_{\text{batch}} = C_{\text{in}} - \int_0^t r dt \quad (15)$$

Danckwerts (1958) and Zwietering (1958) both realised that the residence-time distribution, even that for a continuous stirred tank (CSTR), does not represent complete mixing. Instead, Eq. (14) envisages that following situation: fluid entering the reactor is dispersed into small, but finite, sized elements that remain intact for the duration of their residence in the reactor. The probability of an element exiting the reactor after time  $t$  is

given by the residence time. Since the elements remain intact for the transit through the reactor, each element acts as a batch reactor. The overall exit concentration is then the weighted average of the concentration achieved in each batch, as represented by Eq. (14).

The solution of the normal mass balance, given by Eq. (7) and the segregated-flow model, given by Eq. (14), do not yield the same results for the same parameters, a result of the incomplete state of mixing assumed in Eq. (14). The work by Zwietering (1958) in this regard is seminal.

Zwietering (1958) and Danckwerts (1958) restricted their work to homogeneous systems. On the other hand, Crundwell (1994) derived the equivalent model for heterogeneous systems. The segregated-flow version of the number balance is derived by considering the following situation. Fluid, or slurry, entering the reactor is broken up into small but finite elements that remain intact for the duration of their stay in the reactor. The feed, and each fluid element that is derived from the feed, contains both solid and liquid phases. The probability that an element containing both liquid and particles leaves the reactor after time  $t$  is given by the residence time distribution. Therefore, by analogy with Eq. (14), the number balance for the segregated-flow model is given by

$$\begin{aligned} & \text{(Number of particles leaving reactor in size class } \ell) \\ &= \sum_{\text{All times } t} \text{(Number of particles in size class } \ell \text{ and in} \\ & \quad \text{batch that has spend time } t \text{ in reactor)} \\ & \times \text{(Probability of spending time } t \text{ in reactor)} \end{aligned} \quad (16)$$

or in mathematical terms:

$$n(\ell) = \int_0^\infty n_{\text{batch}}(\ell) E(t) dt \quad (17)$$

The solution to the partial differential equation describing the batch number balance was determined by the method of characteristics (Crundwell, 1994), and is given by

$$n_{\text{batch}}(\ell) d\ell = n_{\text{in}}(L) dL \quad (18)$$

where  $L$  and  $\ell$  are related by

$$t = \int_L^\ell \frac{d\ell}{r_s} \quad (19)$$

The model is completed by Eqs. (10) and (11).

Rearrangement of Eqs. (17)–(19) reveals the leaching number as the only parameter

$$n(\ell^*) = \int_0^\infty n_{\text{batch}}(\ell^*, t^*) E(t^*) dt^* \quad (20)$$

$$n_{\text{batch}}(\ell^*) d\ell^* = n_{\text{in}}(L^*) dL^* \quad (21)$$

$$t^* = \int_{L^*}^{\ell^*} \frac{d\ell^*}{(r_s \bar{t} / \bar{\ell})} = \int_{L^*}^{\ell^*} \frac{d\ell^*}{N_L} \quad (22)$$

As with the population balance, the only parameter is the leaching number.

The segregated-flow model is usually found in terms of the conversion. The substitution of (17) and (18) into Eq. (10) gives

$$1 - X = \frac{\int_0^\infty \int_0^\infty \ell^3 n_{\text{in}}(L) dLE(t) dt}{\int_0^\infty L^3 n_{\text{in}}(L) dL} \quad (23)$$

By noting that  $X_{\text{batch}}(t) = 1 - \ell^3/L^3$ , and by making use of the transformation between mass and number distributions, we can write:

$$1 - X = \int_0^\infty \int_0^\infty (1 - X_{\text{batch}}(t)) m_{\text{in}}(L) dLE(t) dt \quad (24)$$

where  $m_{\text{in}}(L)$  is the mass distribution of the inlet stream.

The segregated-flow model has been called the doubly integrated micro-model or the multiple convolution integral. The naming of these methods has made the discussion unclear for practitioners, since they are not new models, they are merely mathematical formulations the equations describing segregated-flow model. Indeed, the confusion is compounded, since Dixon (1995) derived Eq. (17) from Eq. (24), a case of deriving the model from its rearranged formulation. He called Eq. (17) the multiple convolution integral; however, it is clearly the number balance for the segregated-flow model, as given by Crundwell (1994).

### 3.3. Comparison between the population balance and the segregated-flow models

Experience in other fields of chemical engineering indicates that the segregated-flow model is not appropriate unless operating in conditions of low diffusivity and high viscosity. Since these conditions are not found in leaching, the effects of segregation in the micro-mixing of the slurry phase are not pronounced. Consequently, the population balance is the correct approach.

It may seem that Eq. (24) is easier to compute than the population balance. This is true for a single tank reactor. However, for second and subsequent stages, Eq. (24) cannot be used if the concentration of the lixiviant varies between stages. This is because Eq. (24) carries no information about the particle size distribution on exit. For two or more stages, there is no

difference in computational effort required for the population balance and the segregated-flow model.

The distinction between the population balance and the segregated-flow model is not only an academic matter. Fig. 4 compares the conversion as a function of the leaching number for the segregated model and the population balance for the case in which the mineral and lixiviant, say acid, are fed to the reactor in stoichiometric proportion. The figure indicates that that numerical results differ significantly between the two models. Indeed, if the design engineer required 90% conversion, the segregated-flow model would indicate a tank of half the size of the population balance! The incorrect use of the segregated-flow model could lead to massive under-specification of the process equipment, and consequently, the failure of the processing plant.

There is only one condition under which they give the same numerical result: if the kinetics are zero order, or pseudo-zero order (that is, the concentration in solution is constant because there is a massive excess). This is case is not often found in practice.

The two models of homogeneous and heterogeneous reactors are compared in Table 1.

Independent of the design engineer's personal preference for the population balance or the segregated-flow model, the leaching number is central in both models. The leaching number is the only parameter that can affect the performance of the reactor.

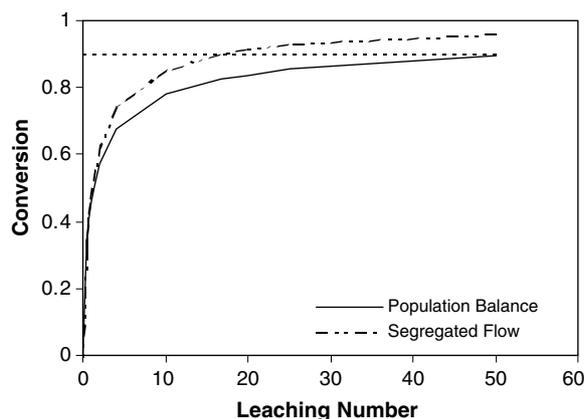


Fig. 4. A comparison of the population balance and the segregated-flow model for the leaching of a mineral by a lixiviant in stoichiometric proportion to mineral. The tank volumes to achieve 90% conversion differ between the two models by more than a factor of 2.5.

Table 1  
Comparison of the segregated-flow model and the population balance

	Well mixed reactor	Segregated-flow reactor
Mixing assumption	Fluid mixed to maximum extent	Fluid divided into elements
Equation for homogeneous reactors	$QC_{\text{in}} = QC_{\text{out}} - rV$	$C_{\text{out}} = \int_0^\infty C_{\text{batch}}(C_{\text{in}}, t) E(t) dt$
Equivalence condition for homogeneous reactors	Equivalent to each other only if the reaction is first order	
Equation for heterogeneous reactors	$Qn_{\text{in}} = Qn_{\text{out}} - V \frac{dr_s n_{\text{out}}}{dt}$	$n(\ell) = \int_0^\infty n_{\text{batch}}(\ell) E(t) dt$
Equivalence condition for heterogeneous reactors	Equivalent to each other only if reaction is zero order	

#### 4. The importance of the leaching number

As an example of the significance of the leaching number, consider the leaching of sulphide concentrates. The three categories of leaching technologies that compete with one another in the technology marketplace are pressure oxidation, leaching after fine milling and bacterial leaching. We will show that the leaching number establishes the theoretical relationship between the technologies.

It was argued earlier that the leaching number indicates that in order to increase the amount of material dissolved, the leaching number must increase. This means that either the residence time,  $\bar{t}$ , or the leaching rate,  $r_s$ , must increase, or the mean particle size in the feed,  $\bar{\ell}$ , must decrease.

Each of these leaching technologies (that is, pressure leaching, bacterial leaching and fine grinding and leaching) balances the leaching number,  $N_L$ , at roughly the same value by changing the values of the parameters of  $N_L$ . These operating regimes are summarised in Table 2.

Thus, the fine grinding and leaching process decreases  $\bar{t}$  and  $\bar{\ell}$ , and increases temperature (which increases  $r_s$  through the activation energy) compared with bacterial leaching. Pressure leaching increases temperature and decreases  $\bar{t}$  compared with bacterial leaching. (In addition, some processes have attempted to increase the intrinsic leaching rate,  $r_s$ , by the addition of agents known to increase the rate of dissolution reactions, such as halide ions.)

Three processes for the leaching of chalcopyrite are compared in Table 3. The BioCOP process, developed by BHP Billiton, is a bacterial leaching process, the AAC/UBC process is a pressure leaching operation at 150 °C, while the Phelps Dodge/Placer Dome process

operates at the harshest conditions. The rate of leaching has been calculated assuming that chalcopyrite dissolves at a rate of  $1 \times 10^{-10}$  m/min at 25 °C and an activation energy of 60 kJ/mol.

In spite of the wide range of temperatures, the leaching number,  $N_L$ , remains in a tight range between 0.5 and 0.7, because trade-offs have been made with regard to the other parameters of the leaching number. It is the leaching number that elucidates how these technologies are related, in spite of wide range of operating variables.

The discussion thus far has focused on the leaching reaction, and assumed that other reactions in the system, such as mass transfer, do not affect the performance of the reactor. In the next section, it is demonstrated under what conditions this holds.

#### 5. The leaching number and the design of continuous leaching reactors

Three leaching situations commonly arise. These three cases will be discussed with a view to using the leaching number to gain insight into the design and operation of these reactors.

##### 5.1. Case 1: direct leaching with a lixiviant

The direct leaching of a mineral with acid or any other lixiviant is chemically the simplest case. For example, consider the leaching of NiO with an acid. The chemical reactions are:



The extent of leaching is described by Eqs. (9)–(11), with  $r_s$  given by

$$r_s = 8 \times 10^9 [\text{H}_2\text{SO}_4] \exp(-80\,000/RT) \quad (26)$$

Since  $r_s$  is not dependent on the particle size, Eq. (26) implies that the rate of dissolution is described by the shrinking-particle model.

Nickel oxide is fed at a rate of 1 t/h to a leaching vessel of 40 m<sup>3</sup>. Acid is added in stoichiometric proportion. The nickel oxide has a particle size distribution described by the Rosin Rammler distribution with the values of the parameters  $p$  and  $q$  equal to 2 and 10, respectively.

The results for this case are shown in Fig. 5. In order to achieve a conversion of 90%, for example, the leaching number should be at least 20. Only the three factors contributing to the leaching number can be adjusted to achieve this. The leaching number can be increased by (i) increasing  $r_s$  by increasing the temperature, (ii) decreasing the mean particle size,  $\bar{\ell}$ , or (iii) increasing the residence time by either increase the vessel volume or decreasing the flow of material to the vessel.

Table 2  
Operating regimes based on the dimensionless leaching number,  $N_L$

Technology	Temperature	Residence time	Particle size	$N_L$
Bacterial leaching	Low	High	Moderate	Moderate
Pressure leaching	High	Low	Moderate	Moderate
Fine grinding and leaching	Moderate	Moderate	Low	Moderate

Table 3  
Process conditions for leaching of chalcopyrite

Technology	Temperature	Particle size, $\mu\text{m}$	Rate, m/min	Residence time in first tank, min	$N_L$
BioCOP	70	10.0	2.3E–09	2880	0.69
AAC/UBC	150	5.0	1.2E–07	20	0.51
Placer Dome/ Phelps Dodge	220	37.0	1.4E–06	15	0.58

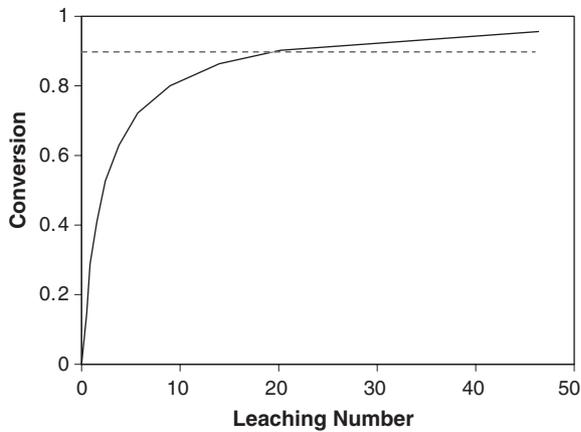
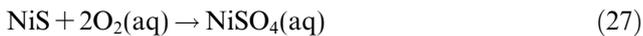


Fig. 5. The effect of the leaching number on the performance of the reactor.

As a result of this knowledge, the economic trade-offs between increased grinding or increased vessel volume can be calculated, and clear decisions made.

### 5.2. Case 2: leaching with a gaseous reactant

A common problem found industrially is the leaching of mineral in which a gas, such as oxygen, chlorine or sulphur dioxide, is the reactant. In this case, the gas must dissolve into solution before it can react with the mineral. For example, consider the leaching of NiS with oxygen. The reactions are as follows:



Eq. (27) represents the dissolution reaction, while, Eq. (28) represents the mass transfer of oxygen from the gas phase to the liquid phase.

The extent of leaching is described by Eqs. (9)–(11), with  $r_s$  given by

$$r_s = 5 \times 10^{12} [\text{O}_2(\text{aq})] \exp(-80000/RT) \quad (29)$$

A mass balance for oxygen is required to complete the model. This mass balance must be consistent with the model chosen. If the segregated model is chosen, Eq. (14) must be used. In this case, the population balance has been chosen, so that the mass balance is given by

$$Q[\text{O}_2(\text{aq})]_0 = Q[\text{O}_2(\text{aq})] - k_L a V ([\text{O}_2]_{\text{sat}} - [\text{O}_2(\text{aq})]) + \frac{r_{\text{NiS}}}{2} \quad (30)$$

where  $Q$  is the volumetric flow rate,  $V$  is the vessel volume,  $k_L a$  is the mass transfer coefficient,  $[\text{O}_2]_{\text{sat}}$  is the value of saturated oxygen at the temperature and pressure of the gas bubble in the vessel, and  $r_{\text{NiS}}$  is the volumetric production rate in  $\text{mol}/\text{m}^3 \text{h}$ , and is given by

$r_{\text{NiS}} = F_{\text{NiS}} X/V$ , where  $F_{\text{NiS}}$  is the molar feed rate of nickel sulphide to the reactor.

Nickel sulphide is fed at a rate of 1 t/h to a leaching vessel of  $40 \text{ m}^3$ . Oxygen is added in stoichiometric proportion. The nickel sulphide has a particle size distribution described by the Rosin Rammler distribution with the values of the parameters  $p$  and  $q$  equal to 2 and 10, respectively.

The results for this case are shown in Figs. 6 and 7. The rate of leaching has same dependency on the leaching number as before. However, Eq. (30) has introduced an additional parameter, the mass transfer coefficient,  $k_L a$ . The effect that this parameter has on the leaching performance is shown in Fig. 7. Interestingly, the results for these conditions indicate that increasing the mass transfer coefficient does not affect the leaching performance as much the leaching number does. The implications of this are that to increase performance, the leaching number must be increased.

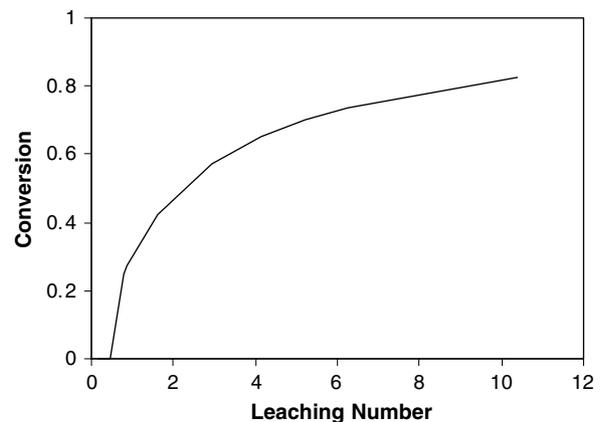


Fig. 6. The effect of the leaching number on the performance of a leaching reactor with a gaseous reactant.

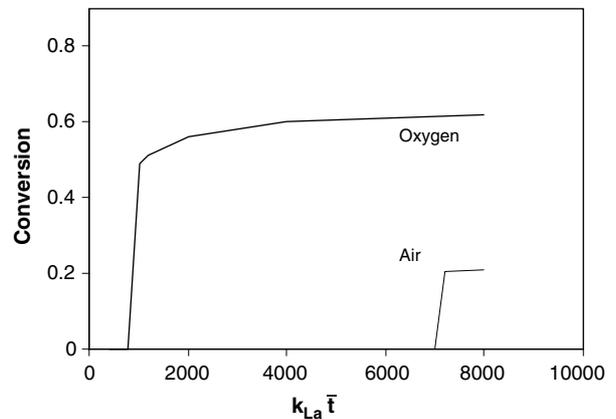
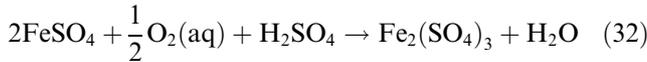
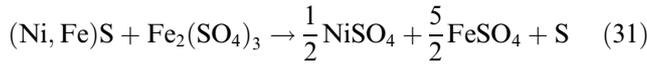


Fig. 7. Effect of the mass transfer coefficient on the performance of the leaching reactor with a gaseous reactant, either 98% oxygen or air.  $\bar{t}$  is the mean residence time.

5.3. Case 3: leaching with ferric sulphate and a gaseous reactant

Another commonly found chemical system is that in which the leaching is performed by ferric ions, and the ferric ions are regenerated by the oxidation of ferrous ions by oxygen. For example, consider the leaching of (Ni, Fe)S in such as system. The chemical reactions are as follows:



The extent of leaching is described by Eqs. (9)–(11), with  $r_s$  given by

$$r_s = 5 \times 10^{10} [Fe_2(SO_4)_3] \exp(-80000/RT) \quad (34)$$

In addition to the leaching reaction, the component balances for oxygen, ferric and ferrous ions must be calculated. These mass balances are given by

$$Q[Fe^{2+}]_0 = Q[Fe^{2+}] + V \left( r_{Fe^{2+}} - \frac{r_{(Ni,Fe)S}}{2.5} \right) \quad (35)$$

$$Q[Fe^{3+}]_0 = Q[Fe^{3+}] + V \left( \frac{r_{(Ni,Fe)S_2}}{2} - r_{Fe^{3+}} \right) \quad (36)$$

$$Q[O_2]_0 = Q[O_2] - k_L a V ([O_2]_{sat} - [O_2]) + \frac{r_{Fe^{2+}}}{4} \quad (37)$$

the volumetric production rate in mol/m<sup>3</sup> h,  $r_{(Ni,Fe)S}$  is given by  $r_{(Ni,Fe)S} = F_{(Ni,Fe)S} X/V$ , where  $F_{(Ni,Fe)S}$  is the molar feed rate of nickel sulphide to the reactor. The rate of oxidation of ferrous ions is given by

$$r_{Fe^{2+}} = 1 \times 10^{16} [FeSO_4]^2 [O_2(aq)] \exp(-8251/T) \quad (38)$$

Nickel sulphide is fed at a rate of 1 t/h to a leaching vessel of 40 m<sup>3</sup>. The nickel sulphide has a particle size distribution described by the Rosin Rammler distribution with the values of the parameters  $p$  and  $q$  equal to 2 and 10, respectively.

The results for this case are shown in Figs. 8–10. Again, it is the leaching number that has the greatest

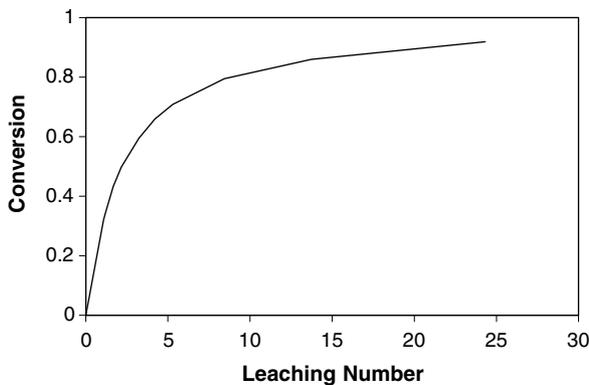


Fig. 8. The effect of the leaching number on the performance of the leaching reactor.

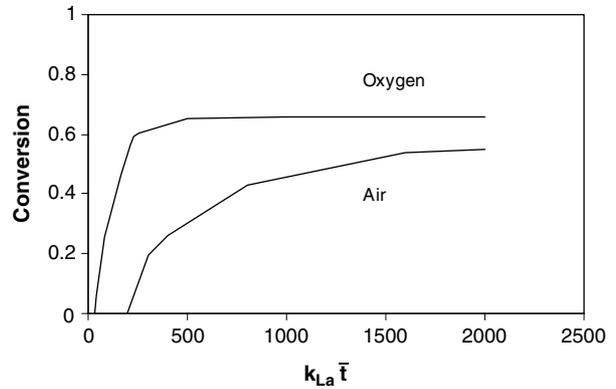


Fig. 9. Effect of the mass transfer coefficient on the performance of the leaching reactor in which a gaseous reactant oxidises a dissolved oxidant, such as ferric ion. Shown are the results for using 98% oxygen and air.  $\bar{t}$  is the mean residence time.

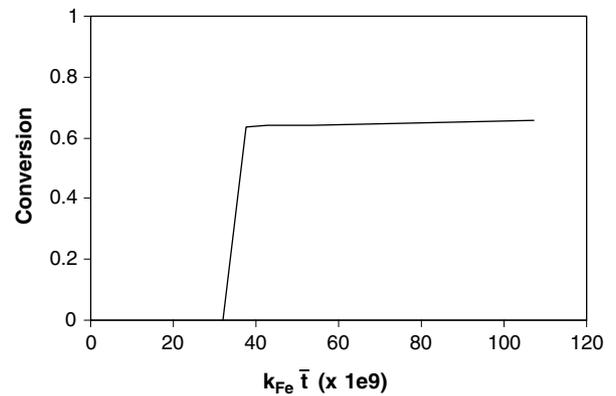


Fig. 10. Effect of the rate of ferrous ion oxidation on the performance of the leaching reactor.  $\bar{t}$  is the mean residence time.

sensitivity, and to increase performance, it must be increased.

Interestingly, increasing the rate of oxidation of ferrous ions does not improve the performance of the reactor. This has practical implications for bacterial leaching, in that the searching for or the breeding of a “super-bug” that has a higher rate of ferrous ion oxidation will not improve the performance of the reactor.

6. Conclusions

Although the segregated-flow model has been used by a number of authors recently, the population balance is the correct method for describing leaching reactors. The distinction between the mathematical approaches is not academic, and can result in significant design errors.

This work has defined the leaching number, has examined its significance in the development of new

leaching technologies, and has examined three common cases of leaching. The leaching number elucidates the effects of leaching rate, particle size and residence time in the performance of leaching reactors.

It has been shown through cases that increasing the mass transfer coefficient or the rate of ferrous oxidation does not enhance the performance of the leaching reactor. Only increasing the leaching number can enhance the performance.

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